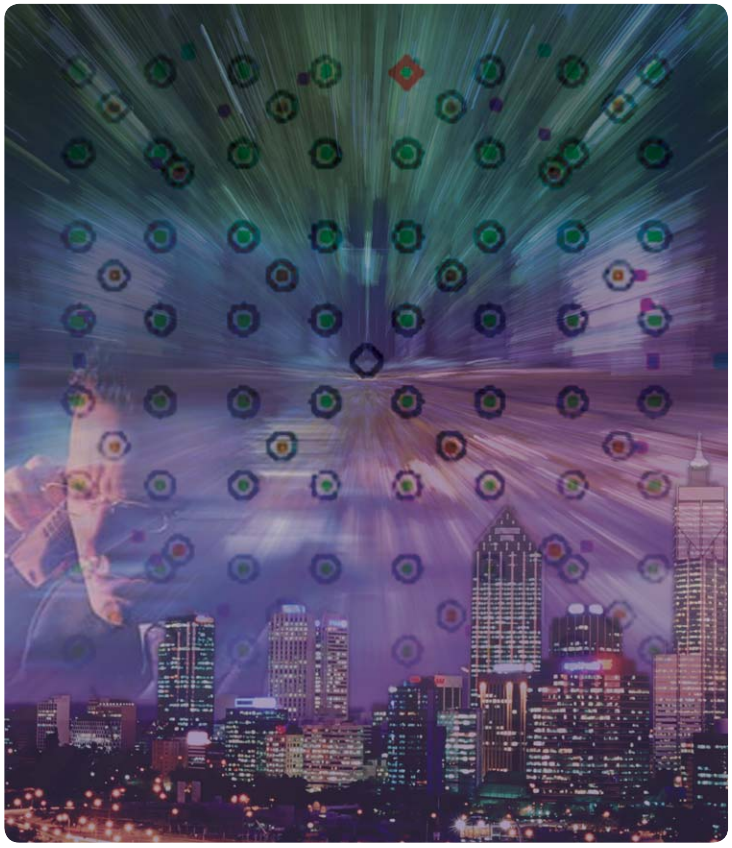
MIMO Performance and Condition Number in LTE Test

Application Note



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Introduction

As companies rush to get Long Term Evolution (LTE) products to market, engineers face tough challenges in testing these often complex devices. The mandate to include Multiple-Input Multiple-Output (MIMO) means many engineers are working to optimize their multi-antenna architectures. To achieve time-to-market goals, the MIMO solutions under development need to work correctly in real-world situations. Defining situations when MIMO transmissions will improve system performance can be complicated, but it is a critical aspect of successful implementation.

Of particular interest is how to quickly determine whether a MIMO channel is capable of supporting spatial multiplexing under a given signal-to-noise ratio (SNR). Fortunately with the proper test equipment, a figure of merit called "condition number" can provide a short-term indication of the SNR required to properly recover a MIMO transmission over the selected wireless channel.

This application note will review the basic concepts of MIMO with specific application to spatial multiplexing in 3rd Generation Partnership Project (3GPP) LTE. It will show measurements of channel coefficients and associated condition numbers as they relate to the SNR at the LTE receiver. How antenna and channel correlation affects system performance will also be discussed, along with recommendations for the best measurement tools to use when developing LTE products and systems.

Multi-antenna wireless systems have been shown to improve data capacity through spatial multiplexing and improve system reliability through antenna diversity and spatial coding such as Space-Time Block Coding (STBC) and Space-Frequency Block Coding (SFBC). The suitability of a MIMO wireless system for spatial multiplexing is largely dependent on the characteristics of the wireless channel, the antenna configuration and the ability of the receiver to accurately recover the channel coupling matrix coefficients. The complexity of the wireless channel including channel correlation, interference and noise rate may create difficulties when measuring the operation and troubleshooting a MIMO system. Fortunately, the operation of a LTE MIMO system with application to spatial multiplexing can be quickly verified with a calculation of the channel matrix "condition number" using a vector signal analyzer (VSA) such as the Agilent 89601A. The condition number is a deterministic calculation for evaluating the performance of the wireless channel and estimating the associated increase in SNR required for successful signal demodulation in the LTE MIMO system. For example, a condition number close to the ideal value of 0 dB would imply perfect channel conditions for the application of spatial multiplexing, while values greater than 10 dB would point to the need for a dB per dB improvement in the relative SNR in order to properly demodulate the MIMO transmission. It can also be shown using the Agilent 89601A VSA that the channel matrix and associated condition number is a function of subcarrier frequency and time which may be a useful tool when studying the effects of banding subcarriers at the evolved Node B (eNB) for the highest performance.

Spatial Multiplexing and MIMO Operation

Spatial multiplexing through MIMO offers an increase in the transmission rate while using the same bandwidth and total transmit power when compared to a traditional Single-Input Single-Output (SISO) system. The theoretical increase in capacity is linearly related to the number of transmit/receive antenna pairs added to the MIMO system, although in practice, spatial multiplexing requires better channel conditions and higher SNR than an equivalent SISO system. Antenna configurations found in the LTE specifications [1] include the two transmit and two receive antenna configuration, generally referred as a "2x2", and the four transmit and four receive antenna configuration, or "4x4". A MIMO system can also be configured with an unequal number of antennas between the transmitter and the receiver, such as the "4x2" configuration, useful for combining spatial multiplexing with antenna diversity. When using an unequal number of antennas between the transmitter and receiver, the MIMO capacity improvement is constrained by the smaller number of antenna ports.

Figure 1 shows a basic 2x2 spatial multiplexing configuration and the associated four complex channel coefficients. The figure also shows an extension to 4 transmit antennas resulting in a 4x2 configuration. The 4x2 system would result in eight channel coefficients. In a typical 2x2 configuration for spatial multiplexing with a single user, the incoming data is multiplexed onto two separate streams or "layers" and simultaneously transmitted from two antennas using the same frequency spectrum. In the uplink, with two users, the independent data streams can be transmitted with one stream from each antenna. This process represents a doubling of the transmitted symbol rate compared to a SISO system, assuming that the overall channel conditions are adequate to fully support the intended MIMO operation. In a 4x4 configuration, theoretically, the capacity would increase by a factor of four.

Data recovery in a spatially multiplexed system requires accurate knowledge of the channel coefficients at the receiver. In a real-world environment, the channel coefficients are affected by correlations and noise in the wireless channel. The channel coefficients may also change rapidly over time as a result of mobility in the transmitter and receiver, or both, and the effects of changing multipath and interference in the surrounding environment. As a result, the receiver or test equipment must be capable of rapid measurement of the channel coefficients. As shown in Figure 1, transmission of the upper data stream from transmit antenna Tx0 to the receive antenna Rx0 occurs over a wireless channel with a complex channel coefficient referred to as h00. Transmission from antenna Tx0 to the antenna Rx1 occurs over a channel with a complex channel coefficient h10. Each channel coefficient represents a composite of all the signal paths, connecting the transmit antenna to the respective receive antenna. There are other coefficients between Tx1 and the two receive antennas resulting in a total of four unique channel coefficients, h00, h10, h01 and h11 for the 2x2 configuration. With proper antenna placement it is desired that these channel coefficients are substantially uncorrelated. Achieving low correlation is non-trivial with practical handheld devices. It can be shown that correlation between channel coefficients will reduce the capacity of a spatially multiplexed MIMO system [2].

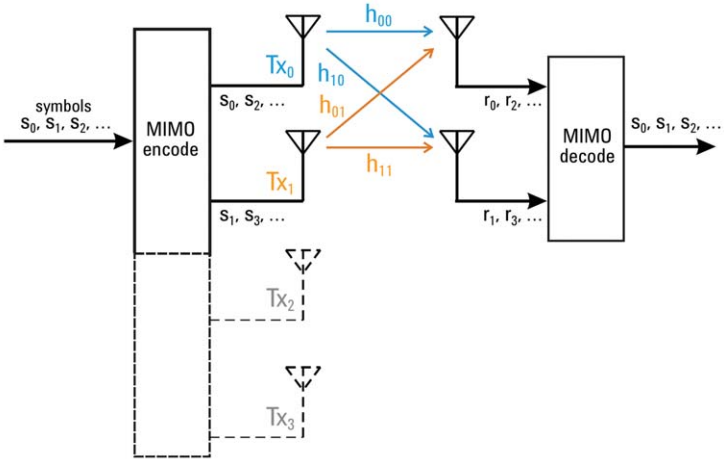

Figure 1. Basic 2x2 MIMO system using spatial multiplexing and having four channel matrix coefficients. Also shown is the option for two additional transmitter antennas creating a 4x2 configuration. Note for the 4x2 case, the channels coefficients are not shown.

In LTE, the channel coefficients are measured using a "non-blind" technique where predefined orthogonal frequency division multiplexing (OFDM) reference signals (RS) are simultaneously transmitted from each antenna. These reference signals use different subcarriers and will be used to directly measure the amplitude and phase response of the channel coefficients. For example, Figure 2 shows the measured spectrograms and frequency responses from two transmit antennas in a LTE 2x2 MIMO system. The measurements were made using Agilent MXA signal analyzers operating with the 89601A VSA software configured for LTE signal demodulation and MIMO analysis. The plots on the left are typical spectrogram measurements showing the frequency content, shown along the x-axis, and time, along the y-axis. The signal amplitude is displayed as color variations with red having highest amplitude and blue have the lowest. The spectrogram measurements show how various portions of the LTE waveform, containing data and reference signals, change over time. Patterns emerge in areas where the reference signals are transmitted as frequency subcarrier locations alternate between the two antennas. For example, in LTE the RS may be transmitted during the 1st and 5th symbol times. In the spectrogram plots of Figure 2, the location of the RS symbols are noted for the two transmissions, Tx0 and Tx1. The figure also shows the time-gated spectrum of the RS in the right column. Comparing the two plots on the right, it can be clearly observed that the frequency subcarriers on the upper plot, transmitted from Tx0, are orthogonal (use different subcarriers) to the subcarriers transmitted from Tx1, shown on the lower right plot. During this portion of the LTE waveform, the receivers can measure these individual subcarriers to determine the channel coefficients associated with transmissions from Tx0 and Tx1. It is important to note that for wideband signals, the channel coefficient's response may vary across the transmitted frequency spectrum; therefore, in a typical OFDM-MIMO scheme, there will be a set of channel coefficients measured at each subcarrier frequency.

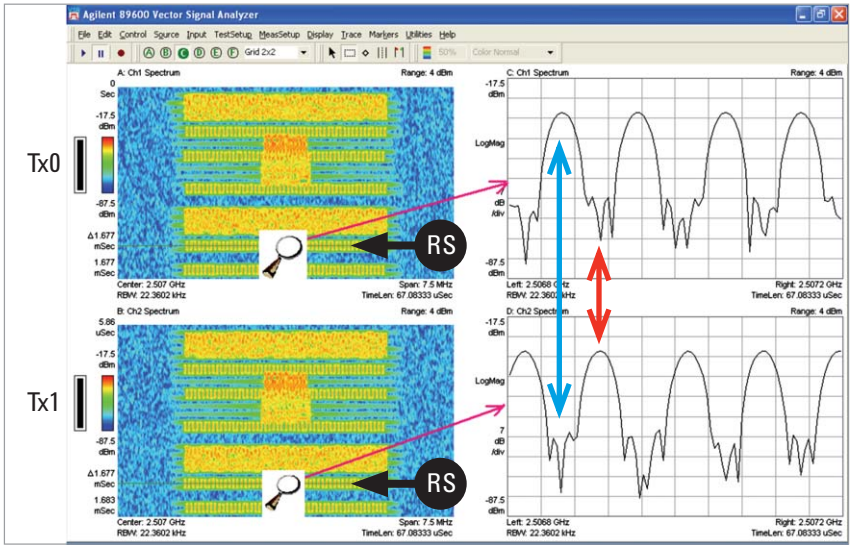

Figure 2. Measurements of the spectrograms and frequency responses for a LTE 2x2 MIMO signal. The upper spectrogram measurement, shown on the left, is for the signal transmitted from antenna Tx0 and the lower plot from antenna Tx1. The time-gated spectrums, shown on the right, display the orthogonal RS (pilot) subcarriers used for estimating the channel coefficients. The measurements were made using the Agilent 89601A VSA using a Hanning window with a time constant set to match the symbol length.

Continuing with the 2x2 MIMO system shown in Figure 1, after the four channel coefficients are determined, the upper antenna, Rx0, will receive the data signal r0 being a combination of the transmitted data streams from Tx_0 and Tx_1 modified by the channel coefficients h_{00} and h_{01} respectively. At the same time, the lower antenna receives the signal r_1 as a combination of the transmitted streams modified by the channel coefficients, h_{10} and h_{11} respectively. Over the first symbol time (as shown in Figure 1) the received signals, r_0 and r_1, are a function of transmitted data symbols, s_0 and s_1, and the channel coefficients. Mathematically, these received signals can be represented as shown in Equation 1.

$$r_0 = h_{00}s_0 + h_{01}s_1$$
$$r_1 = h_{10}s_0 + h_{11}s_1$$

Equation 1.

These equations correspond to a system of linear equations and may be described in matrix form as shown in Equation 2. Note that Equation 2 also includes the effects of noise added to the received signals.

$$[r]=[H][s]+[n]$$

Equation 2.

Under favorable channel conditions, the spatial signatures of the two signals, r_0 and r_1, are uncorrelated and therefore easily separated.

The receiver, using the RS to acquire knowledge of the channel coefficients, can recover the first two symbols, s_0 and s_1. As the received signal includes noise, the accuracy in recovering the transmitted signals, s_0 and s_1, is directly related to the SNR at each receiver and how accurately the coefficients in the channel matrix, [H], were previously measured. The quality of the channel matrix is affected by multipath, antenna correlation, noise and interference in the wireless channel. A measurement of the matrix quality can be provided by a calculation of the condition number, K(H) for the channel matrix [H].

Condition Number

A system of linear equations is "well-conditioned" if small errors in the matrix coefficients result in small errors in the solution. A system of linear equations is "ill-conditioned" when small errors in the coefficients may have a large detrimental effect on the solution. Often with round-off errors, finding a solution using an ill-conditioned matrix may require the use of double or triple precision in order to reach a specific accuracy in the final solution. The accuracy in the solution may also decrease when the measurements of the matrix coefficients are corrupted by high levels of noise and interference, such as the case when measuring the RS pilot subcarriers in a LTE MIMO transmission with low SNR. There are several techniques to quantify the matrix properties including a key parameter referred as the matrix "condition number". The condition number is a traditional and deterministic calculation formed by taking the ratio of the maximum to minimum singular values of the matrix. For MIMO, the channel condition number, K(H), is calculated from the instantaneous channel matrix [H] without the need for stochastic averaging. Small values for the condition number imply a well-conditioned channel matrix while large values indicate an ill-conditioned channel matrix. The condition number can provide an indication as to whether the MIMO channel is capable of supporting spatial multiplexing and is a short-term indication of the SNR required to properly recover a MIMO transmission.

In MIMO analysis, the condition number for the channel matrix, K(H), is often expressed as a numerical value in dB. A well-conditioned matrix will have approximately equal singular values and the condition number will be low and typically less than 10 dB. The ideal condition number, K(H), is equal to one, or 0 dB. When K(H) is greater than 10 dB, data recovery at the receiver becomes progressively more sensitive to errors in the measured channel coefficients resulting from additive white Gaussian noise (AWGN), quantization noise and other interference present in the system. It can be shown that the sensitivity of the recovered data, $\frac{\|\Delta s\|}{\|s\|}$, is related to the sensitivity of the received signal, $\frac{\|\Delta r\|}{\|r\|}$, and the value of condition number, K(H), by the relationship shown in Equation 3 [3]. Here $\|r\|$ represents the vector norm of the associated column vector for the transmitted and received data.

$$\frac{\|\Delta s\|}{\|s\|} \le K(H)\frac{\|\Delta r\|}{\|r\|}$$

Equation 3

Table 1. Comparison of a well-conditioned and ill-condition channel matrix.

Channel matrix [H]	Singular values	Condition number
$\begin{bmatrix}1.0 & 0.1\\ 0.2 & 0.9\end{bmatrix}$	$\begin{bmatrix}1.1094\\ 0.7832\end{bmatrix}$	1.4 (2.9 dB)
$\begin{bmatrix}1.0 & 0.9\\ 0.5 & 0.9\end{bmatrix}$	$\begin{bmatrix}1.6726\\ 0.2690\end{bmatrix}$	6.2 (15.9 dB)

Thus, when the condition number is large, a small error in the received signal may result in large errors in the recovered data. On the other hand, when the condition number is small and approaching an ideal linear value of 1, the system is not any more affected by noise than a traditional SISO system. Two simple examples of a channel matrix and associated condition number for a 2x2 MIMO system are shown in Table 1. The channel matrix coefficients in these examples are real-valued but in practice the coefficients would generally be complex numbers having magnitude and phase components. The upper row in Table 1 shows the channel matrix for a well-conditioned system. By taking the ratio of the associated singular values, 1.11 and 0.79, the condition number is calculated as 1.4 or 2.9 dB. As this condition number is less than 10 dB, it is can be assumed that this well-conditioned channel matrix would be useful when recovering the data streams in a MIMO system. For the next row in Table 1, the channel matrix has singular values of 1.6726 and 0.2690. The associated condition number is calculated as 6.2 or 15.9 dB. In this case, the condition number is greater than 10 dB and it would be expected that data recovery at the receiver would be very sensitive to noise and measurement errors in that system.

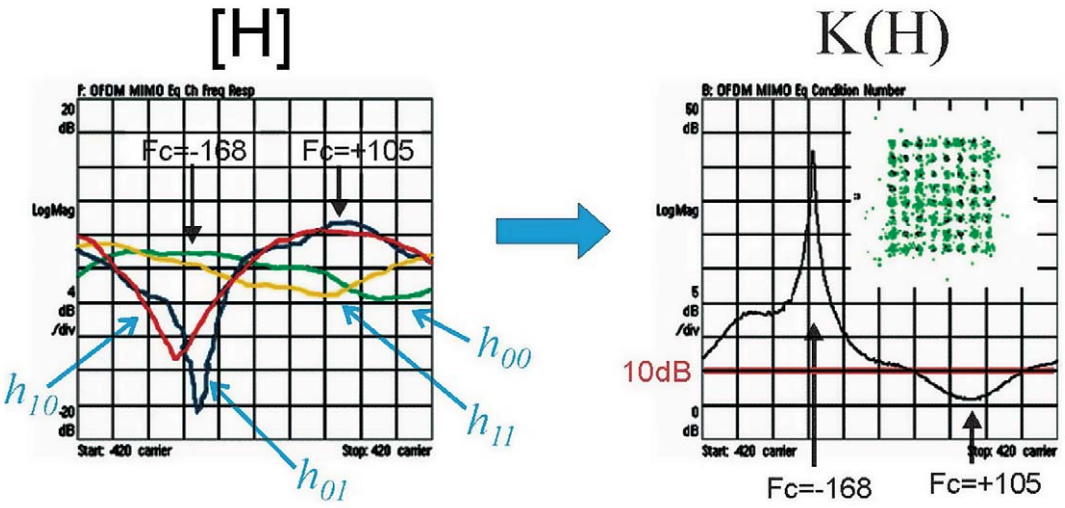

Figure 3. Measurements of the four channel coefficients and associated condition number as a function of subcarrier frequency for an LTE 2x2 MIMO signal. Note the measured channel responses are normalized while the condition number is calculated from the complex channel coefficients before normalizing.

The condition number of a matrix can be measured directly from an LTE MIMO signal using the Agilent 89601A VSA software. As an example, Figure 3 shows the measured normalized frequency response of the four channel coefficients (on the left) and the associated condition number (on the right) for a 2x2 MIMO signal operating in a multipath environment. When examining the magnitude of the channel coefficients as a function of subcarrier frequency, if all the responses have a similar amplitude, the displayed measurements do not provide any particular insight into whether the channel matrix is well-conditioned or not. Using the VSA to calculate and display the associated condition number, it can be easily determined which subcarriers have a well-conditioned channel matrix and which do not. For the example shown in Figure 3, at subcarrier frequency equal to +105, the condition number has a low value of 6 dB. For this subcarrier, it is expected that the measured channel would be appropriate for MIMO spatial multiplexing. On the other hand, for subcarrier frequencies near -168, the condition numbers peak well over 10 dB. Over this portion of the subcarriers, it may be difficult to properly demodulate the associated symbols due to the low quality of the channel matrix. To improve system performance, LTE allows grouping or banding of those subcarriers yielding the highest MIMO performance. It is important to note that the condition numbers will change with variations in the channel characteristics that will likely occur over time.

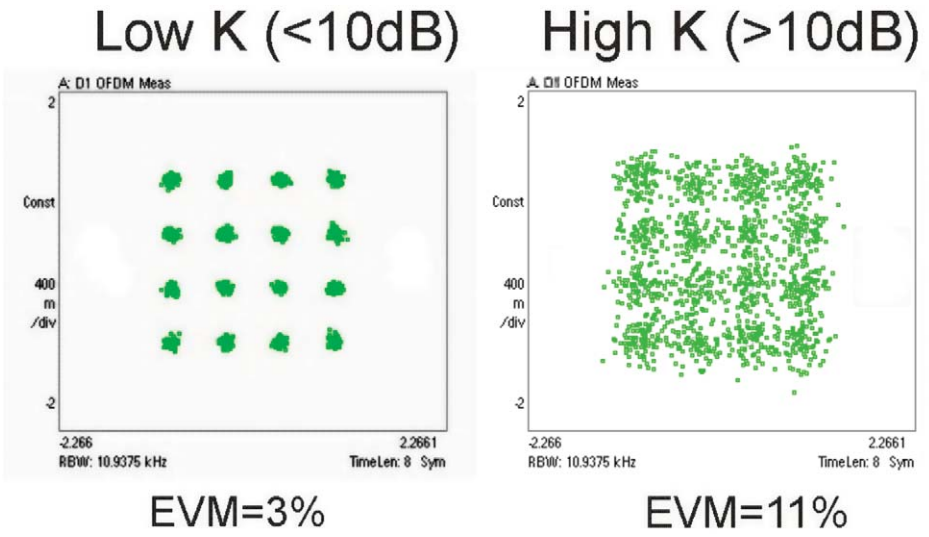

Figure 4. Measured constellation and EVM performance for an LTE 2x2 MIMO system having low and high condition numbers.

As a comparison of how a poorly conditioned channel matrix can affect the demodulation characteristics of a MIMO signal, Figure 4 shows the constellation plots for two different channels with one having an overall low condition number and the other having a high number. The measurement on the left in Figure 4 shows a 16QAM constellation having a tightly clustered distribution of symbols around the ideal constellation points. The associated condition numbers for this MIMO channel were less than 10 dB across all subcarriers. In this system, the measured error vector magnitude (EVM) is 3 percent. In comparison, the measured constellation on the right in Figure 4 displays a much wider distribution of symbols resulting in an 11 percent EVM. For this measurement, the associated condition number as a function of frequency was previously shown in Figure 3.

In the case of Figure 3, the high condition numbers over a large portion of the frequency range is the likely cause for the poor constellation and low EVM performance. Just viewing the constellation, it looks the same as if the signal has a poor SNR. You cannot tell directly if SNR is the problem, or if it is due to the channel coefficients. When troubleshooting MIMO components and systems, it is important to examine the condition number in order to understand how a poorly formed channel may affect the measured results. The condition number can also provide troubleshooting insight even when the channel is simply a cabled connection.

The condition number is a composite result calculated from measurements taken across all receiver antennas. As previously shown, when the measured condition number is high, due to poor channel conditions, it may be difficult to properly recover the MIMO signal. One obvious technique to improve system performance is to increase the SNR across the multiple receivers. It is known that the SNR requirements are higher in a MIMO system in order to achieve higher capacity while maintaining the same level of EVM performance relative to a single input, single output (SISO) system. In this case, the condition number can be used to estimate the increase in SNR required in MIMO as compared to SISO. Figure 5 shows the relationship between condition number and the relative SNR required to achieve a fixed level of system performance. For example, if the measured condition number is 20 dB, the SNR in the MIMO system would need to be 13 dB higher than the SISO system to achieve the same level of performance. If the condition number is less than 5 dB, the relative SNR improvement required would be approximately 1-2 dB. The curve shown in Figure 5 is an idealized case having the same SNR for each data layer at all receiver antennas. In reality, the SNR would vary for each layer and asymmetry may exist between the layers at the multiple receivers, thus the relative SNR may need to be even higher in an actual MIMO system. When asymmetries exist in a MIMO signal, it may be possible to equalize the layer performance at the receivers using a technique called "precoding" which is included as part of the LTE specification [1]. Precoding cross-couples the layers prior to transmission into the wireless channel with the goal of equalizing the performance across the multiple receive antennas.

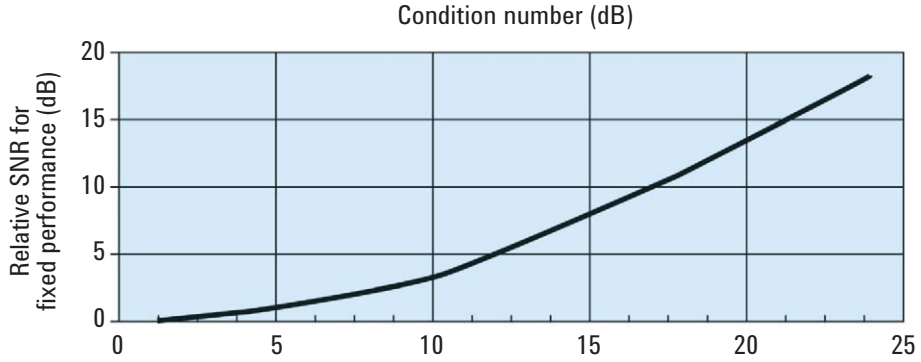

Figure 5. Empirical relationship between the MIMO condition number and the relative SNR required to achieve the same level of performance in MIMO and SISO systems.

As shown in Figure 5, there is a vital relationship between the required SNR and the condition number when comparing the SNR requirements for a MIMO system. Another way to examine the effects of the channel and SNR is to compare the performance of the same MIMO system under different levels of SNR and channel conditions. A MIMO system with poor channel conditions and high SNR may have similar performance to this system operating with an ideal channel and lower SNR. For example, Figure 6 shows a comparison of several measured constellations for a 2x2 MIMO system operating under different SNR and channel conditions. Figure 6a shows the measured constellation for a MIMO system when the SNR is 30 dB and the channel conditions are ideal. These conditions may be difficult to achieve in practice but provides a good baseline for comparison. In this case, the measured EVM using the Agilent 89601A VSA software is approximately 3 percent. When the channel conditions degrade and the associated condition number increases, it is expected that the system performance will be reduced. For this case, as shown in Figure 6b, poor channel conditions result in the high condition number of 15 dB, the deterioration of the measured constellation and the increase of the associated EVM to 10 percent. This MIMO channel was simulated with a static –3 dB cross coupling between the two transmit channels and the SNR was fixed at 30 dB. As a third case, the same system operating under ideal channel conditions, as in Figure 6a, but with a reduced SNR, again results in a 10 percent EVM. The resulting constellation is shown in Figure 6c. For this measurement, the SNR is set to approximately 20 dB. When comparing the measurements in Figures 6b and 6c, there is not an absolute dB to dB relationship between condition number and SNR with the system operating at the same performance, but it is apparent that both the MIMO channel and the SNR will be important factors when determining the overall performance in LTE MIMO system. Since the condition number shows exactly how SNR needs to improve in order to allow spatial multiplexing, it is a useful static test that can be easily implemented by a receiver designer.

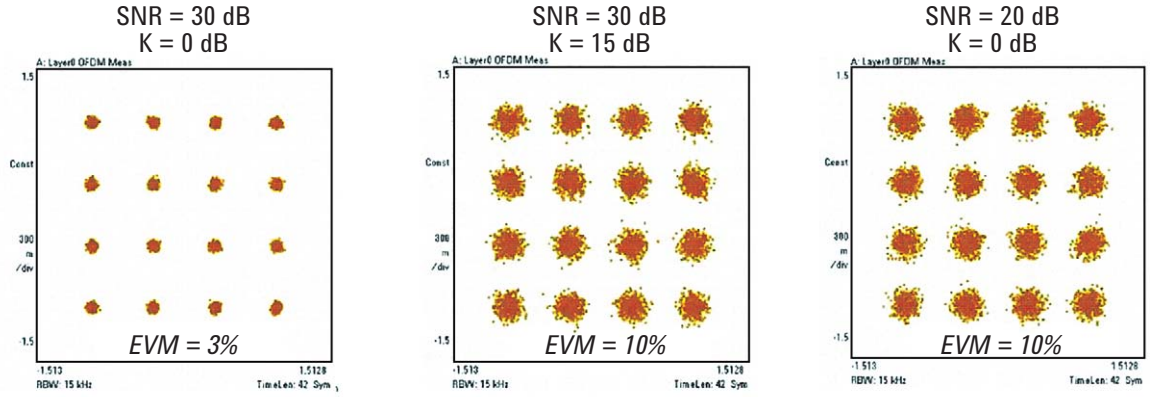

Figure 6a Figure 6b Figure 6c

Figure 6. Measured constellation and associated EVM for a 2x2 MIMO system operating under various SNR and channel conditions. For Figure 6a, the SNR equals 30 dB and channel is ideal. For Figure 6b, the SNR is 30 dB and condition number is 15 dB. For Figure 6c, the SNR is 20 dB and channel is ideal.

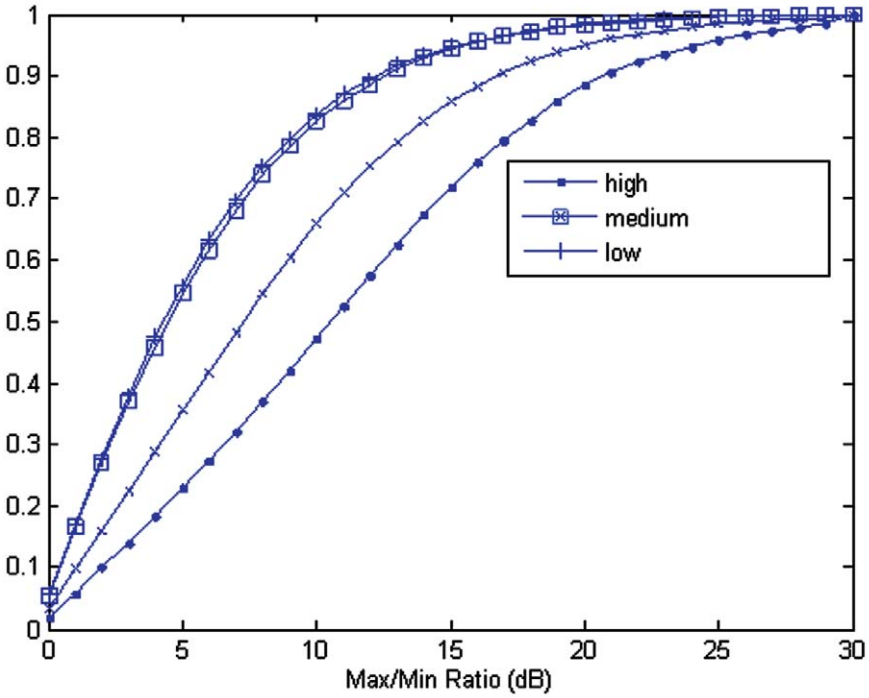

Figure 7. Cumulative Density Function (CDF) for the condition number as a function of correlation between the channel coefficients.

It was previously discussed that an ill-conditioned MIMO channel matrix will result in a system prone to error and highly sensitive to noise and interference. Correlation in the channel matrix coefficients may also result in an ill-conditioned matrix and the associated condition number will also be large. Correlation between the matrix coefficients are the result of one or more of the following: inadequate antenna spacing, common antenna polarization and narrow angular spread created by the surrounding environment [2]. A MIMO system is more likely to have a lower condition number when the channel coefficients have low correlation. Correlation is measured statistically for many modeled channels, so there is not a direct relationship. Figure 6 shows the cumulative density function (CDF) of the condition number in a MIMO system having low, medium and high correlation between antenna elements. The curves shown in Figure 7 were obtained using the Agilent N5106A PXB MIMO receiver tester for introducing varying amounts of antenna correlation into the 2x2 MIMO system.

As an example, there is approximately a 50 percent probability that the condition number will be 5 dB or less when the channel coefficients have a low correlation, but the probability is reduced to 20 percent when the coefficients are highly correlated. Therefore, it is desirable to design the MIMO system with low correlation between channel coefficients by using widely spaced antenna elements or antenna elements that are cross-polarized.

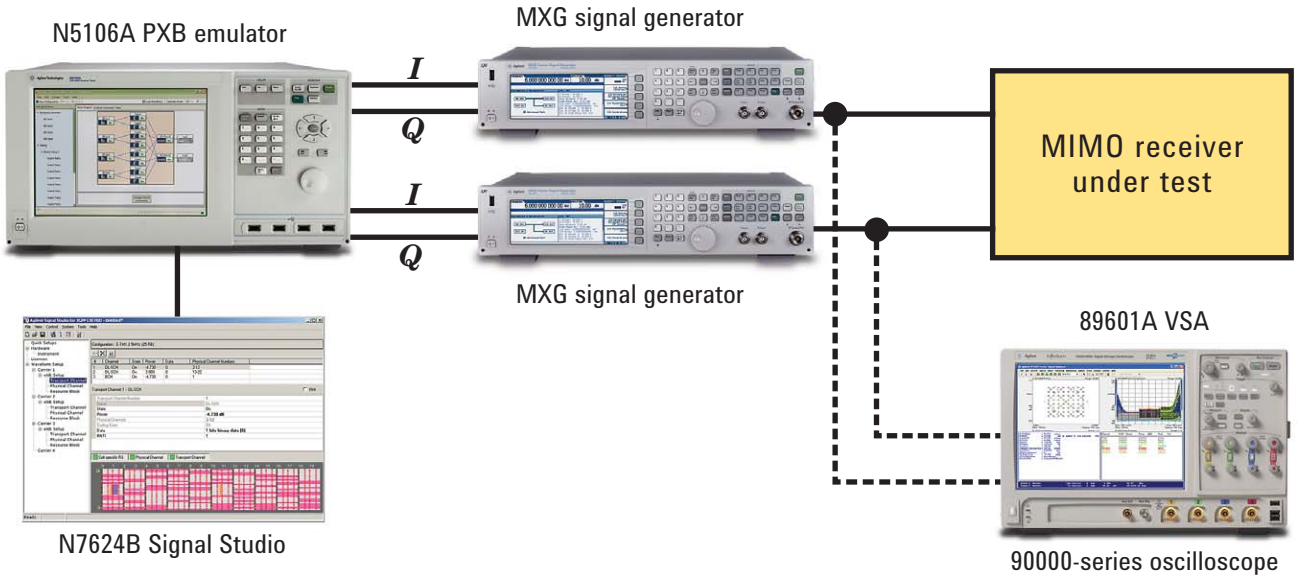

Figure 8. Measurement configuration for testing the performance of a 2x2 MIMO receiver under a variety of multipath conditions.

A typical configuration for measuring the performance of a 2x2 MIMO receiver is shown in Figure 8. The 3GPP LTE signal is generated using the Agilent N7624B Signal Studio software, and the waveform files are downloaded into the Agilent N5106A PXB internal baseband generators. The N5106A PXB channel emulator and baseband generator is used to replicate real-world MIMO conditions including multipath fading, noise, interference and channel correlations. The N5106A PXB outputs the baseband I and Q signals for upconversion by a pair of Agilent MXG-series RF signal generators. The I and Q waveforms from the PXB can be connected to the MXG generators over an analog or digital bus. The upconverted signals from the MXG generators are connected to the MIMO receiver under test. To verify the LTE signal performance, an Agilent 90000-series oscilloscope configured with 89601A software can be substituted for the actual MIMO receiver. The 89601A VSA can test the quality of the signal applied to the MIMO receiver under a variety of multipath conditions.

Conclusion

It has been shown that the matrix condition number is a very useful quantitative tool for determining the overall quality of the LTE MIMO channel. The condition number can also be used to estimate the required increase in SNR in a spatially-multiplexed MIMO system when compared to a traditional SISO system. This knowledge can help receiver designers understand how well a MIMO receiver should be able to recover a signal under particular channel conditions, and design tests for their receivers. As shown in Figure 6, knowledge of the condition number can help provide insight to the quality of the channel that would not otherwise be obvious.

References

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